Mean-field model of jet formation in a collapsing Bose-Einstein condensate

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Abstract.

We perform a systematic numerical study, based on the time-dependent Gross-Pitaevskii equation, of jet formation in collapsing and exploding Bose-Einstein condensates as in the experiment by Donley *et al.* [2001 Nature **412** 295]. In the actual experiment, via a Feshbach resonance, the scattering length of atomic interaction was suddenly changed from positive to negative on a pre-formed condensate. Consequently, the condensate collapsed and ejected atoms via explosion. On a disruption of collapse by suddenly changing the scattering length to zero a radial jet of atoms was formed in the experiment. We present a satisfactory account of jet formation under the experimental conditions as well as make predictions beyond experimental conditions which can be verified in future experiments.

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1. Introduction

Recent successful observation of Bose-Einstein condensates (BEC) of trapped alkali atoms has initiated the intensive study of different novel phenomena. On the theoretical front, numerical simulation based on the time-dependent nonlinear mean-field Gross-Pitaevskii (GP) equation [1] is well under control and has provided a satisfactory account of some of these phenomena.

Since the detection of BEC of ^{7}Li atoms with attractive interaction, one problem of extreme interest is the dynamical study of the formation and decay of BEC for attractive atomic interaction [2]. In general a attractive condensate larger than a critical size is not dynamically stable [2]. However, if such a condensate is "prepared" or somehow made to exist it experiences a dramatic collapse and explodes emitting atoms.

A dynamical study of the collapse has been performed by Donley et al. [3] on an attractive ^{85}Rb BEC [4] in an axially symmetric trap, where they manipulated the inter-atomic interaction by changing the external magnetic field exploiting a nearby Feshbach resonance [5]. In the vicinity of a Feshbach resonance the atomic scattering length a can be varied over a huge range by adjusting the external magnetic field. Consequently, they changed the sign of the scattering length, thus transforming a repulsive condensate of ^{85}Rb atoms into an attractive one which naturally evolves into a collapsing and exploding condensate. Donley et al. have provided a quantitative estimate of the explosion of this BEC by measuring different properties of the exploding condensate.

It has been realized that many features of the experiment by Donley *et al.* [3] can be described by the mean-field GP equation [6–17]. However, we are fully aware that there are features of this experiment which are expected to be beyond mean-field description. Among these are the distribution of number and energy of emitted high-energy ($\sim 10^{-7}$ Kelvin) uncondensed burst atoms reported in the experiment. Although there have been some attempts [9–11] to describe the burst atoms using the mean-field GP equation, now there seems to be a consensus that they cannot be described adequately and satisfactorily using a mean-field approach [12–17]. The GP equation is supposed to deal with the zero- or very low-energy condensed phase of atoms and has been successfully used to predict the time to collapse, evolution of the collapsing condensate as well as its oscillation [7–9, 11, 12]. However, the GP equation has not been fully tested to study the very low-energy ($\sim$ nano Kelvin) jet formation [3] when the collapse is suddenly stopped before completion by jumping the scattering length to zero (noninteracting atoms) or positive (repulsive atoms) values. As the jet atoms are very low-energy condensed atoms the mean-field GP equation seems to be suitable for their study and we present such a systematic description in this paper.

In the experiment [3] the initial scattering length a_{initial} (≥ 0) of a repulsive ($a > 0$) or noninteracting ($a = 0$) condensate is suddenly jumped to the negative value a_{collapse} (< 0) to start the collapse. The condensate then begins to collapse and explode. The collapse is then suddenly terminated after an interval of time t_{evolve} by jumping the scattering length from a_{collapse} to a_{quench} (≥ 0). The jet atoms are

slowly formed in the radial direction when the collapse is stopped in this fashion. In the experiment usually $a_{\rm quench} = 0$. Sometimes the scattering length is jumped past $a_{\rm quench}$ to $a_{\rm expand} = 250a_0$ to have an expanded condensate to facilitate observation. It is emphasized that unlike the emitted uncondensed "hotter" missing and burst atoms reported in the experiment [3] the jet atoms form a part of the surviving "colder" condensate and hence should be describable by the mean-field GP equation. Yet most of the theoretical treatments on the topic [7, 8, 11–15] are completely silent about jet formation. Saito *et al.* [9] and Bao *et al.* [17] present a mean-field description of jet formation and Calzetta *et al.* [16] goes beyond the mean-field model in including the effects of quantum field corrections in their description of jet formation. Bao *et al.* [17] employ a fully asymmetric mean-field model and are capable of studying the breakdown of axial symmetry in jet formation for experiments of collapse performed in an axially symmetric trap [3].

Although, the breakdown of axial symmetry and the possible necessity of quantum field corrections in the jet formation are interesting topics to be studied carefully, we investigate the possibility of explaining the jet formation within an axially-symmetric mean-field model. Using the GP equation we study satisfactorily the jet formation for the experimental values of the scattering lengths and times which shows that a mean-field model describes the essential features of jet formation. The number of jet atoms is in agreement with experiment. Further, we extend our study to other values of scattering lengths and times and predict the possibility of the formation of jet atoms. Future experiments may test these predictions and thus provide a more stringent test for the mean-field GP equation. To account for the loss of atoms from the strongly attractive collapsing condensate we include an absorptive nonlinear three-body recombination term in the GP equation [6].

In section 2 we present our mean-field model. In section 3 we present our results that we compare with the experiment and other numerical studies. In section 4 we present a physical discussion of our findings and some concluding remarks are given in section 5.

2. Nonlinear Gross-Pitaevskii Equation

The time-dependent Bose-Einstein condensate wave function $\Psi(\mathbf{r};\tau)$ at position $\mathbf{r}$ and time τ allowing for atomic loss may be described by the following mean-field nonlinear GP equation [1]

$$\begin{aligned}&\Big[-i\hbar\frac{\partial}{\partial \tau} - \frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) + gN|\Psi(\mathbf{r};\tau)|^2 - \frac{i\hbar}{2} \\ &\times \left(K_2 N|\Psi(\mathbf{r};\tau)|^2 + K_3 N^2 |\Psi(\mathbf{r};\tau)|^4\right)\Big]\Psi(\mathbf{r};\tau) = 0. \end{aligned} \tag{2.1}$$

Here m is the mass and N the number of atoms in the condensate, $g = 4\pi\hbar^2 a/m$ the strength of inter-atomic interaction, with a the atomic scattering length. The terms K_2 and K_3 denote two-body dipolar and three-body recombination loss-rate coefficients,

respectively and include the Bose statistical factors 1/2! and 1/3! needed to describe the condensate. The trap potential with cylindrical symmetry may be written as $V(\mathbf{r}) = \frac{1}{2}m\omega^2(r^2 + \lambda^2 z^2)$ where ω is the angular frequency in the radial direction r and $\lambda\omega$ that in the axial direction z. The normalization condition of the wave function is $\int d\mathbf{r}|\Psi(\mathbf{r};\tau)|^2 = 1$. Here we simulate the atom loss via the most important quintic three-body term K_3 [6–9]. The contribution of the cubic two-body loss term [20] is expected to be negligible [6,9] compared to the three-body term in the present problem of the collapsed condensate with large density and will not be considered here.

In the absence of angular momentum the wave function has the form $\Psi(\mathbf{r};\tau) = \psi(r,z;\tau)$. Now transforming to dimensionless variables defined by $x = \sqrt{2}r/l$, $y = \sqrt{2}z/l$, $t = \tau\omega$, $l \equiv \sqrt{\hbar/(m\omega)}$, and

$$\phi(x,y;t) \equiv \frac{\varphi(x,y;t)}{x} = \sqrt{\frac{l^3}{\sqrt{8}}}\psi(r,z;\tau), \tag{2.2}$$

we get

$$\Big[-i\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} - \frac{\partial^2}{\partial y^2} + \frac{1}{4}\left(x^2 + \lambda^2 y^2 - \frac{4}{x^2}\right)$$
$$+8\sqrt{2}\pi n \left|\frac{\varphi(x,y;t)}{x}\right|^2 - i\xi n^2 \left|\frac{\varphi(x,y;t)}{x}\right|^4 \Big]\varphi(x,y;t) = 0, \tag{2.3}$$

where $n = Na/l$ and $\xi = 4K_3/(a^2 l^4 \omega)$. The normalization condition of the wave function becomes

$$\mathcal{N}_{\text{norm}} \equiv 2\pi \int_0^\infty dx \int_{-\infty}^\infty dy |\varphi(x,y;t)|^2 x^{-1}. \tag{2.4}$$

For $K_3 = 0$, $\mathcal{N}_{\text{norm}} = 1$, however, in the presence of loss $K_3 > 0$, $\mathcal{N}_{\text{norm}} < 1$. The number of remaining atoms N in the condensate is given by $N = N_0\mathcal{N}_{\text{norm}}$, where N_0 is the initial number of atoms.

In this study the term K_3 will be used for a description of atom loss in the case of attractive interaction. Near a Feshbach resonance the variation of K_3 vs. scattering length a is very rapid and complicated [18]. From theoretical [19] and experimental [20] studies it has been found that for negative a, K_3 increases rapidly as $|a|^n$, where the theoretical study [19] favors $n = 2$ for smaller values of $|a|$. In this work we represent this variation via a quadratic dependence: $K_3 \sim a^2$. This makes the only "parameter" ξ of the present model a constant for an experimental set up with fixed l and ω and in the present study we use a constant ξ. In our previous and the present investigation we choose ξ or K_3 to provide a correct evolution of the number of atoms in the condensate during collapse and explosion. The mean-field GP equation is best-suited to make this prediction. After a small experimentation it is found that $\xi = 2$ fits the time evolution of the condensate in the experiment of Donley *et al.* [3] satisfactorily for a wide range of variation of initial number of atoms and scattering lengths [7]. This value of ξ is used in all simulations reported in this paper. A similar philosophy is used in choosing the value of K_3 in [12,17], where the authors reproduced the rate of variation of the number of atoms in the collapsing condensate. However, interest in other mean-field [9,11] and

beyond-mean-field [13–15] studies of the collapsing condensate was not in reproducing the variation of the number of atoms in the collapsing condensate. Consequently, if a three-body recombination term is used in these studies other criteria are used in fixing the value of K_3.

It is useful to compare this value of $\xi(=2)$ with the experimental [20] estimate of three-body loss rate of ^{85}Rb as well as with other values used in the study of the experiment by Donley *et al.* For this we recall that the present value $\xi = 2$ with $K_3 = \xi a^2 l^4 \omega/4$ leads to [7, 8] $K_3 \simeq 8 \times 10^{-25}$ cm^6/s at $a = -340a_0$ and $K_3 \simeq 6 \times 10^{-27}$ cm^6/s at $a = -30a_0$. Santos *et al.* [11] employed the experimental value [20] $K_3 \simeq 7 \times 10^{-25}$ cm^6/s at $a = -340a_0$ which is very close to the present choice. (Santos *et al.* quote $L_3 \equiv 6K_3$. We overlooked this fact in [7, 8] and hence misquoted there the K_3 values of [11, 20].) Bao *et al.* [17] employed $K_3 \simeq 6.75 \times 10^{-27}$ cm^6/s at $a = -30a_0$ in close agreement with the present choice, which leads to a very similar time evolution of the number of atoms in the collapsing condensate. Savage *et al.* [12] employed a slightly larger value $K_3 \simeq 19 \times 10^{-27}$ cm^6/s at $a = -30a_0$ and also produced reasonably similar results for the time evolution of the number of atoms. However, Saito *et al.* [9] and Duine *et al.* [14] employed a much smaller value (smaller by more than an order of magnitude) $K_3 \simeq 2 \times 10^{-28}$ cm^6/s at $a = -30a_0$. With this value of K_3, unlike in the other studies [7, 11, 12, 17], it is not possible to fit the the time evolution of the number of atoms in the collapsing condensate. Of these theoretical studies, the K_3 values used by Santos *et al.* [11], Savage *et al.* [12], Bao *et al.* [17] and the present author [7] are consistent with each other and describes well the decay of the collapsing condensate.

3. Numerical Result

We solve the GP equation (2.3) numerically using a time-iteration method based on the Crank-Nicholson discretization scheme elaborated in [21]. We discretize the GP equation using time step $\Delta = 0.001$ and space step 0.05 for both x and y spanning x from 0 to 15 and y from -40 to 40. This domain of space was sufficient to encompass the whole condensate wave function in this study.

The calculation is performed with the actual parameters of the experiment by Donley *et al.* [3], e. g,, the initial number of atoms, scattering lengths, etc. The numerical simulation using (2.3) with a nonzero $\xi(=2)$ immediately yields the remaining number of atoms in the condensate after the jump in scattering length. The remaining number of atoms vs. time for $a_{\rm initial} = 7a_0$, $a_{\rm collapse} = -30a_0$, $\xi = 2$, and $N_0 = 16000$ is in satisfactory agreement with experiment [7].

Now we consider the jet formation as in the experiment for these sets of the parameters after different evolution times $t_{\rm evolve}$ of the collapsing condensate when the scattering length is suddenly changed from $a_{\rm collapse} = -30a_0$ to $a_{\rm quench} = 0$ or to $250a_0$. First we consider $a_{\rm quench} = 0$. In figures 1 (a) and (b) we plot the contour plot of the condensate for $t_{\rm evolve} = 4$ ms and 8 ms, respectively, in this case

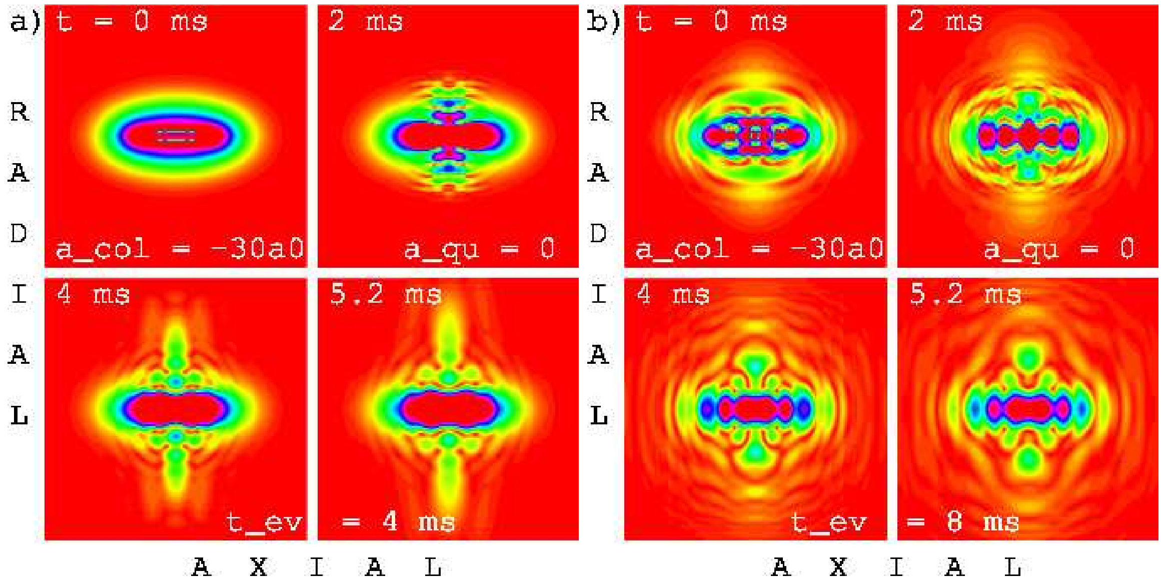

Figure 1. A view of the evolution of radial jet at times $t = 0$, 2ms, 4ms and 5.2 ms on a mat of size 16 μm $\times$ 16 μm from a contour plot of $|\phi(x,y,t)|^2$ for $a_{\mathrm{initial}} = 7a_0$, $a_{\mathrm{collapse}} = -30a_0$, $\xi = 2$, $N_0 = 16000$, $a_{\mathrm{quench}} = 0$, and (a) $t_{\mathrm{evolve}} = 4$ ms and (b) $t_{\mathrm{evolve}} = 8$ ms.

at different times $t = 0$, 2 ms, 4 ms, and 5.2 ms after jumping the scattering length to $a_{\mathrm{quench}} = 0$. A prominent radial jet is formed at time $t = 5.2$ ms after stopping the collapse for $t_{\mathrm{evolve}} = 4$ ms. The jet is formed slowly after stopping the collapse and is more prominent $4-6$ ms after stopping the collapse. The jet for $t_{\mathrm{evolve}} = 8$ ms is not so spectacular. We also studied the jet formation for $t_{\mathrm{evolve}} = 2$ ms, 6 ms, and 10 ms. The jet is much less pronounced for $t_{\mathrm{evolve}} = 2$ ms and 10 ms compared to the jet in figures 1.

Next we study the effect of the variation of $a_{\mathrm{collapse}} = -30a_0$ on the jet formation. For this purpose for the same set of parameters of figures 1 we consider $a_{\mathrm{collapse}} = -6.7a_0$ and $a_{\mathrm{collapse}} = -250a_0$ corresponding to smaller and larger attraction, in figures 2 (a) and (b), respectively. First, we consider the case $a_{\mathrm{collapse}} = -6.7a_0$ in figure 2 (a). In this case the final attraction is weaker and the collapse is less dramatic. The collapse and the decay of atoms do not start until $t_{\mathrm{evolve}} \approx 6$ ms. For $t_{\mathrm{evolve}} \approx 8$ ms, a broad jet is formed after about 12 ms of disruption of collapse. After 4 ms of disruption of collapse there is almost no visible jet. The jets are broader and less prominent for larger values of t_{evolve}. Next we consider the highly attractive case $a_{\mathrm{collapse}} = -250a_0$ in figure 2 (b). In this case due to a very large attraction the collapse and the decay of atoms start at a small value of t_{evolve} close to zero. Hence a reasonable jet is formed for $t_{\mathrm{evolve}} = 2$ ms at small times after stopping the collapse. Because of large attraction in this case the collapse is over for a smaller value of t_{evolve}. Hence there is almost no jet formation for $t_{\mathrm{evolve}} \geq 4$ ms. However, the nature of jets in each case of figures 1 and 2 is distinct.

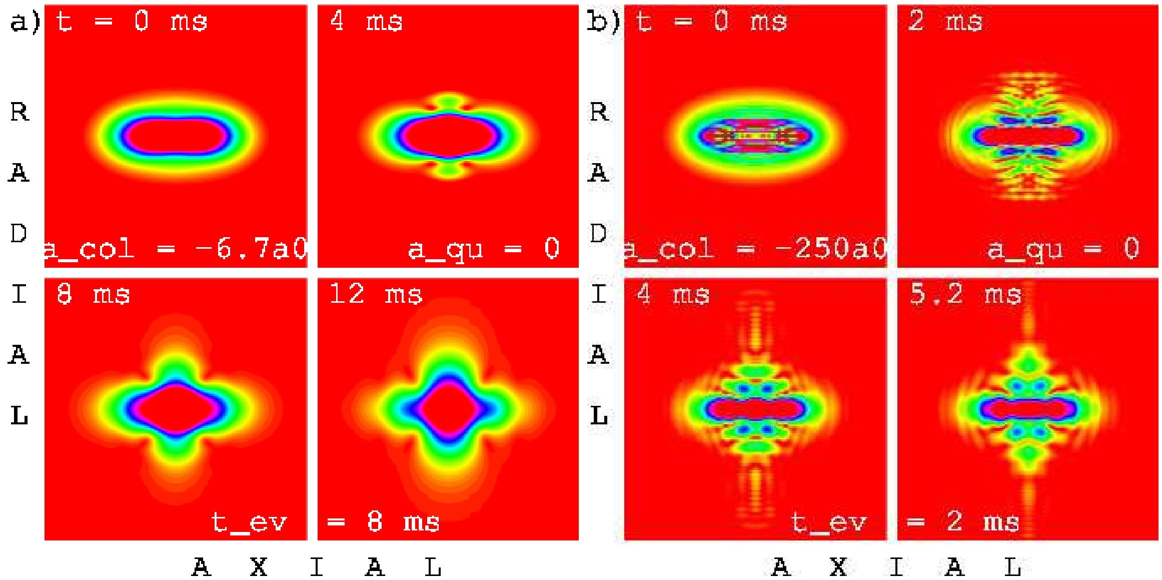

Figure 2. A view of the evolution of radial jet on a mat of size 16 μm $\times$ 16 μm from a contour plot of $|\phi(x, y, t)|^2$ for $a_{\text{initial}} = 7a_0$, $\xi = 2$, $N_0 = 16000$, $a_{\text{quench}} = 0$, and (a) $a_{\text{collapse}} = -6.7a_0$, $t_{\text{evolve}} = 8$ ms, at times $t = 0$, 4ms, 8ms and 12 ms and (b) $a_{\text{collapse}} = -250a_0$, $t_{\text{evolve}} = 2$ ms, at times $t = 0$, 2ms, 4ms and 5.2 ms.

Donley *et al.* [3] also considered expanding the condensate before observing the jet by jumping the scattering length to a large positive value $a_{\text{expand}} = 250a_0$ after disrupting the collapse. This procedure expands the size of the condensate so that it might be easier (or the only way) to observe the jets in the laboratory. However, we find that in all cases the jet is much less pronounced after this expansion. This is illustrated in figures 3 (a) and (b) where we plot the jet formation corresponding to the cases reported in figures 1 (a) and (b), respectively, after expanding the condensate to $a_{\text{expand}} = 250a_0$ as in the experimental result reported in figure 5 of [3]. In plots of figures 3 (a) and (b) the condensate is of larger size than in the corresponding plots of figures 1 (a) and (b). In figure 3 (a) the jet is almost destroyed. In figure 3 (b) the jet appears but it is wider due to expansion. We also expanded to $a_{\text{expand}} = 250a_0$ the jets for $a_{\text{collapse}} = -6.7a_0$ for different t_{evolve}; the jets were almost destroyed in those cases. After expanding to $a_{\text{expand}} = 250a_0$ the jets for $a_{\text{collapse}} = -250a_0$ became wider due to expansion but remained visible.

In addition, we studied jet formation for different values of a_{initial} in place of $a_{\text{initial}} = 7a_0$ and find that the scenario remains very similar independent of the initial scattering length. However, the number of particles in the jet gives a quantitative measure of jet formation and in the following we make a study of the number of atoms in the jet in different cases.

Next we calculated the number of jet atoms in different cases by integrating the wave function over the relevant region where the jet is formed. The normalization condition (2.4) gives the total number of atoms in the condensate via $N_0\mathcal{N}_{\text{norm}}$. After

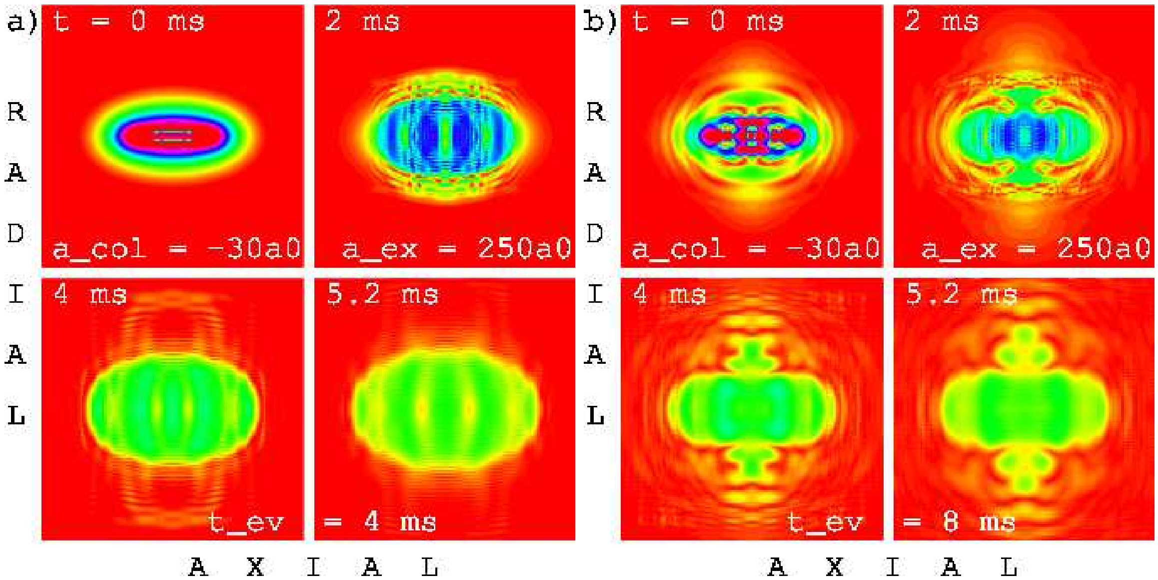

Figure 3. A view of the evolution of radial jet at times $t = 0$, 2ms, 4ms and 5.2 ms on a mat of size 16 μm $\times$ 16 μm from a contour plot of $|\phi(x, y, t)|^2$ for $a_{\text{initial}} = 7a_0$, $a_{\text{collapse}} = -30a_0$, $\xi = 2$, $N_0 = 16000$, $a_{\text{expand}} = 250a_0$, and (a) $t_{\text{evolve}} = 4$ ms and (b) $t_{\text{evolve}} = 8$ ms.

an examination of figures 1 and 2 or other relevant jet figures we separate the condensate at 5.2 ms (as in the experiment) according to x values into the central part and jet. The x integral in (2.4) is then separated into the central part and jet in each case. The jet part (outer x values) of the integral (2.4) multiplied by N_0 gives the number of jet atoms. A similar procedure has been used in [17] to calculate the number of jet atoms. However, in actual experiment to see jet or any other phenomenon, the harmonic trap has to be removed and the condensate allowed to expand and photographed. This enlarges the condensate to be photographed without presumably losing its actual shape and characteristics. Vortices and dark and bright solitons photographed in this fashion give the true picture of the condensate before free expansion. In numerical simulation, on the other hand, it is possible and much easier to count the jet atoms more accurately without any expansion. Assuming that there is not much experimental error in counting the jet atoms after free expansion we attempt to compare the two in the following.

For a fixed $N_0 = 16000$ and $a_{\text{initial}} = 7a_0$ we calculate the number of atoms in the jet for different evolution time t_{evolve} and a_{collapse}. In none of the cases an expansion to a_{expand} was applied. The results are plotted in figures 4. In figure 4 (a) we plot the variation of the number in jet vs. t_{evolve} for $a_{\text{collapse}} = -30a_0$ and in figure 4 (b) we plot the variation of the number in jet vs. $|a_{\text{collapse}}|/a_0$ for $t_{\text{evolve}} = 4$ ms. In figure 4 (a) we also plot the experimental result for the number of atoms in jet for the same values of the parameters and find the agreement of our calculation with experiment to be quite satisfactory. It is noted that an expansion to a_{expand} was not applied in the data reported in figure 6 (a) of [3, 22].

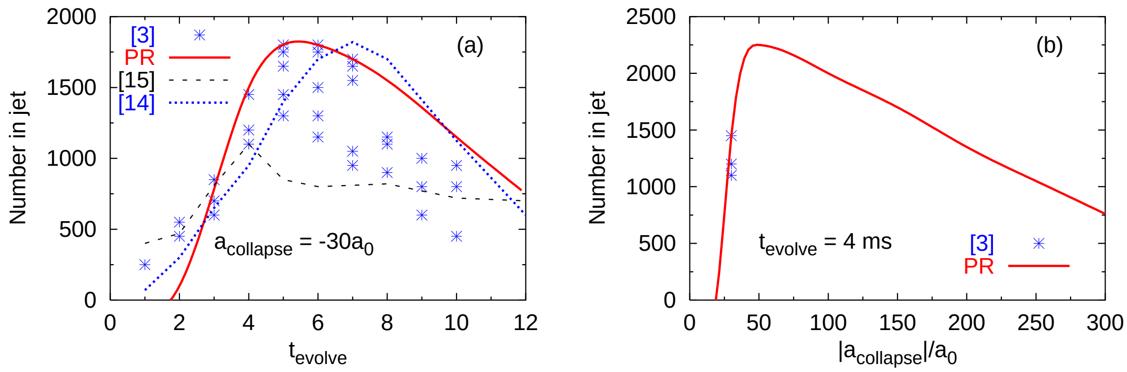

Figure 4. Number of atoms in jet 5.2 ms after jumping the scattering length to $a_{\rm quench} = 0$ for $N_0 = 16000$, $a_{\rm initial} = 7a_0$, $\xi = 2$ vs. (a) $t_{\rm evolve}$ for $a_{\rm collapse} = -30a_0$ and (b) $|a_{\rm collapse}|/a_0$ for $t_{\rm evolve} = 4$ ms; blue star – experiment of Donley *et al.* [3], black dashed line – mean field model of Bao *et al.* [17], blue dotted line – average over theoretical results of Calzetta *et al.* [16], red full line – present result.

In addition, in figure 4 (a) we compare the present results with the theoretical calculations by Calzetta *et al.* [16] and Bao *et al.* [17]. The calculation by Bao *et al.* [17] is essentially based on mean-field GP equations as in this study and they correctly identify the jet atoms as being a part of the condensate. They employ a fully asymmetric mean-field model in their description of jet formation. Yet the present result for the number of atoms in the jet is larger than theirs and in better agreement with experiment [3]. The reason for this discrepancy is unknown.

The calculation by Calzetta *et al.* [16] uses a theoretical model beyond mean-field taking into consideration quantum fluctuations. The number of jet atoms of Calzetta *et al.* [16] is in good agreement with the experiment [3] as well as with the present study. In figure 4 (a) we plot the average over the number of jet atoms obtained by Calzetta *et al.* As the physical inputs and the dynamics of the study of Calzetta *et al.* and the present study are quite distinct, it is difficult to compare the two and conclude about the effect of quantum fluctuations on jet formation. The effect of quantum fluctuations could turn out to be significant in various aspects of the experiment of the collapse of a BEC including the formation of jet atoms. For example, they are of utmost relevance in the molecule formation in a collapsing BEC near a Feshbach resonance and in subsequent rapid atom-molecule oscillation as observed recently [23]. Also, the burst atoms cannot be properly described by a mean-field model and quantum corrections could be significant [16, 17]. Further studies are needed to identify clearly the effect of quantum fluctuations on jet formation.

The investigation by Saito *et al.* [9] is very similar to this study in applying an axially-symmetric mean-field model and producing the essentials of jet formation. They also provide a physical explanation of jet formation. In the collapsing condensate two distinct spikes are formed in the condensate wave function along the axial direction as

the atomic interaction is changed from repulsive to attractive. These two spikes act as sources of matter waves and jet is the interference pattern of matter waves from these two sources [9]. However, we feel that small local spike(s) in the wave function expand in the radial direction in the form of a jet when the collapse is stopped by removing the atomic attraction and the force on the spike(s) is suddenly changed [3].

4. Discussion

In this section we give a physical explanation of the different types of jet formation noted in last section. The jet formation is more dramatic when the vigorous collapse of a condensate is suddenly stopped by turning the attractive condensate noninteracting ($a_{\rm quench} = 0$) or repulsive ($a_{\rm expand} = 250a_0$). In the strongly collapsing condensate local radial spikes are formed during particle loss as can be seen from a plot of the numerically calculated wave function [7] and in experiment [3]. During particle loss the top of the spikes are torn and ejected out and new spikes are formed until the explosion and particle loss are over. There is a balance between central atomic attractive force and the outward kinetic pressure. If the attractive force is now suddenly removed by stopping the collapse by applying $a_{\rm quench} = 0$, the highly collapsed condensate expands due to kinetic pressure, becomes larger and the recombination of atoms is greatly reduced. Consequently, the spikes expand and develop into a prominent jet [3] for $a_{\rm quench} = 0$ as in figure 1 (a).

However, if the condensate is expanded further by applying $a_{\rm expand} = 250a_0$, in the cases studied, the spike as well as the condensate expand so much that the prominent jet becomes in general more diffuse as in figure 3 (b) or completely destroyed as in figure 3 (a). If the attractive condensate is allowed to collapse for sufficiently long time, the explosion stops eventually and a relatively cold remnant condensate is formed. At that stage there would be almost no prominent spikes in the wave function and no jet could be formed by applying $a_{\rm quench} = 0$. With the increase of evolution time of a collapsing condensate the jet becomes less prominent as can be seen in figures 1 (a), (b) and 4 (a) and eventually disappears for large $t_{\rm evolve}$. However, the collapse and decay of particles start after a finite $t_{\rm evolve}$ before which the jet formation is practically absent as in figure 4 (a). If the condensate is weakly attractive as in figure 2 (a) the collapse is also weak and the spikes are less pronounced. Consequently, upon stopping the collapse a wide jet is formed after a longer interval of time as in figure 2 (a). As $|a_{\rm collapse}|$ is increased at a fixed $t_{\rm evolve} = 4$ ms as in figure 4 (b) one gradually passes from a strongly collapsing condensate to a relatively cold remnant, that is from a region of prominent jet formation to a region with less prominent jet. For smaller $|a_{\rm collapse}|$ the collapse is weaker and jet formation is absent as in figure 4 (b).

In the actual experiment [3] the jet is found not to possess axial symmetry. In the present study we use an axially symmetric model to describe the essential features of the jet. Hence, although an axially symmetric model is enough for a qualitative description of the jet, a full three-dimensional model might be necessary for its complete quantitative

description.

5. Conclusion

In conclusion, we have employed a numerical simulation based on the accurate solution [21] of the mean-field Gross-Pitaevskii equation with a cylindrical trap to study the jet formation as observed in the recent experiment of an attractive collapsing condensate by Donley *et al.* [3]. In the GP equation we include a quintic three-body nonlinear recombination loss term [6] that accounts for the decay of the strongly attractive condensate. The result of the present simulation is in good agreement with the experimental result for jet formation [3]. We also compare the present result with two other recent theoretical calculations of jet formation [16, 17]. Of the different aspects of the experiment by Donley *et al.* the dynamics of relatively hot emitted burst and missing atoms seems to be beyond mean-field treatment [16, 17]. However, the various properties of the cold residual condensate including jet formation seem to be describable by mean-field models. In fact, many features of the experiment by by Donley *et al.* [3], specially the detailed behavior of the surviving remnant condensate [7, 9, 17] and jet formation, have been understood by introducing the rather conventional three-body recombination loss in the standard mean-field GP equation, with a loss rate compatible with other studies [7, 12, 17, 18, 20].

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